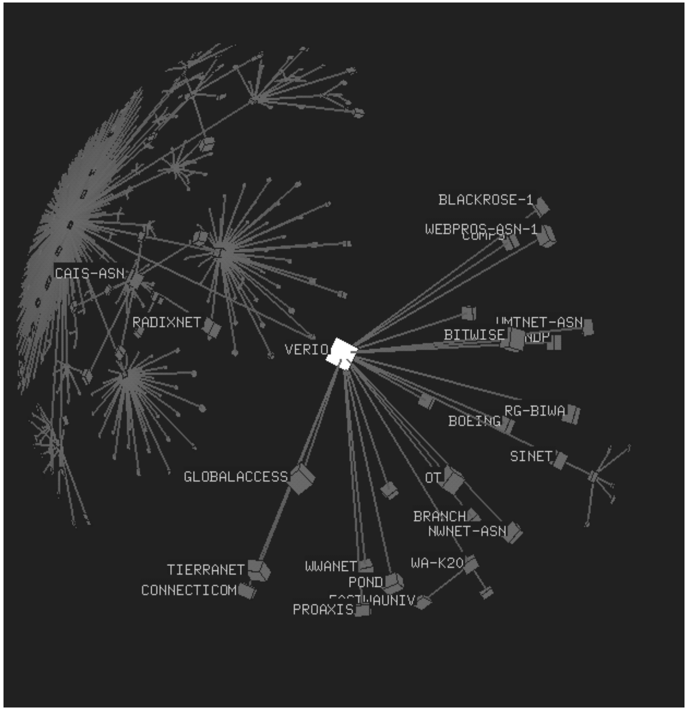
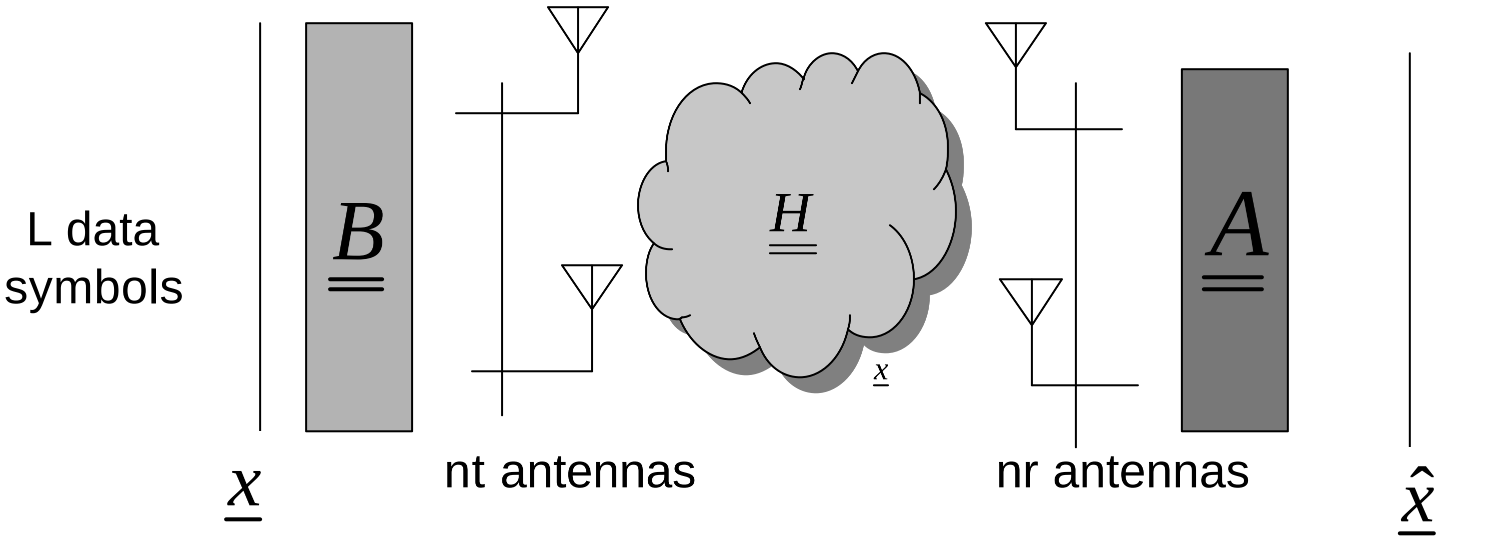
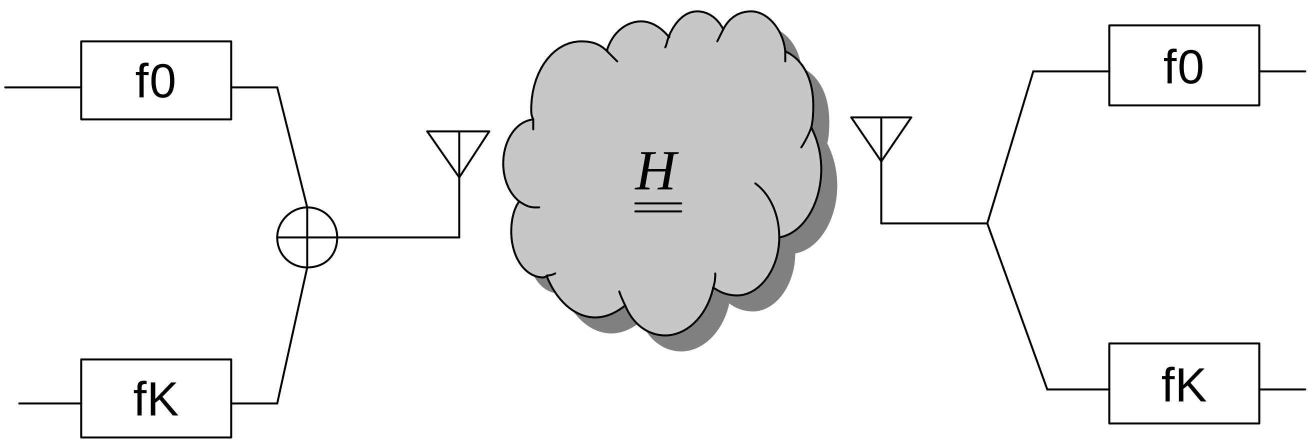
Design alternatives in MIMO channels with CSIT



Miguel Ángel Lagunas
CTTC (Centre for Telecommunications Tecnology of Catalunya)
m.a.lagunas@cttc.es http::/www.cttc.es





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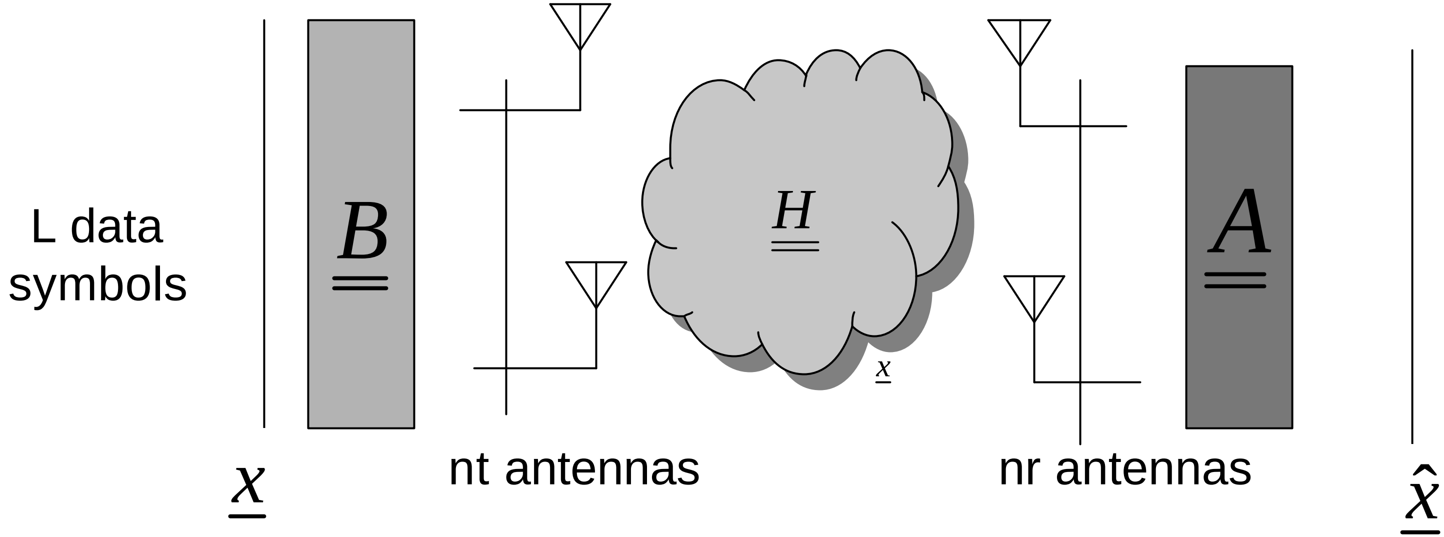


- Frequency and/or space diversity processing.
- A problem of power allocation (bit allocation)
- More symbols than actual channels

$$\underline{\hat{x}} = \underline{\underline{A}}^{H} . \underline{\underline{H}} . \underline{\underline{B}} . \underline{x} + \underline{n}$$

Multiple beamforming (OFDM)

$$\hat{x} = \underline{a}^{H} . \underline{\underline{H}} . \underline{b} . x + n$$

Single beamforming (FDSS)

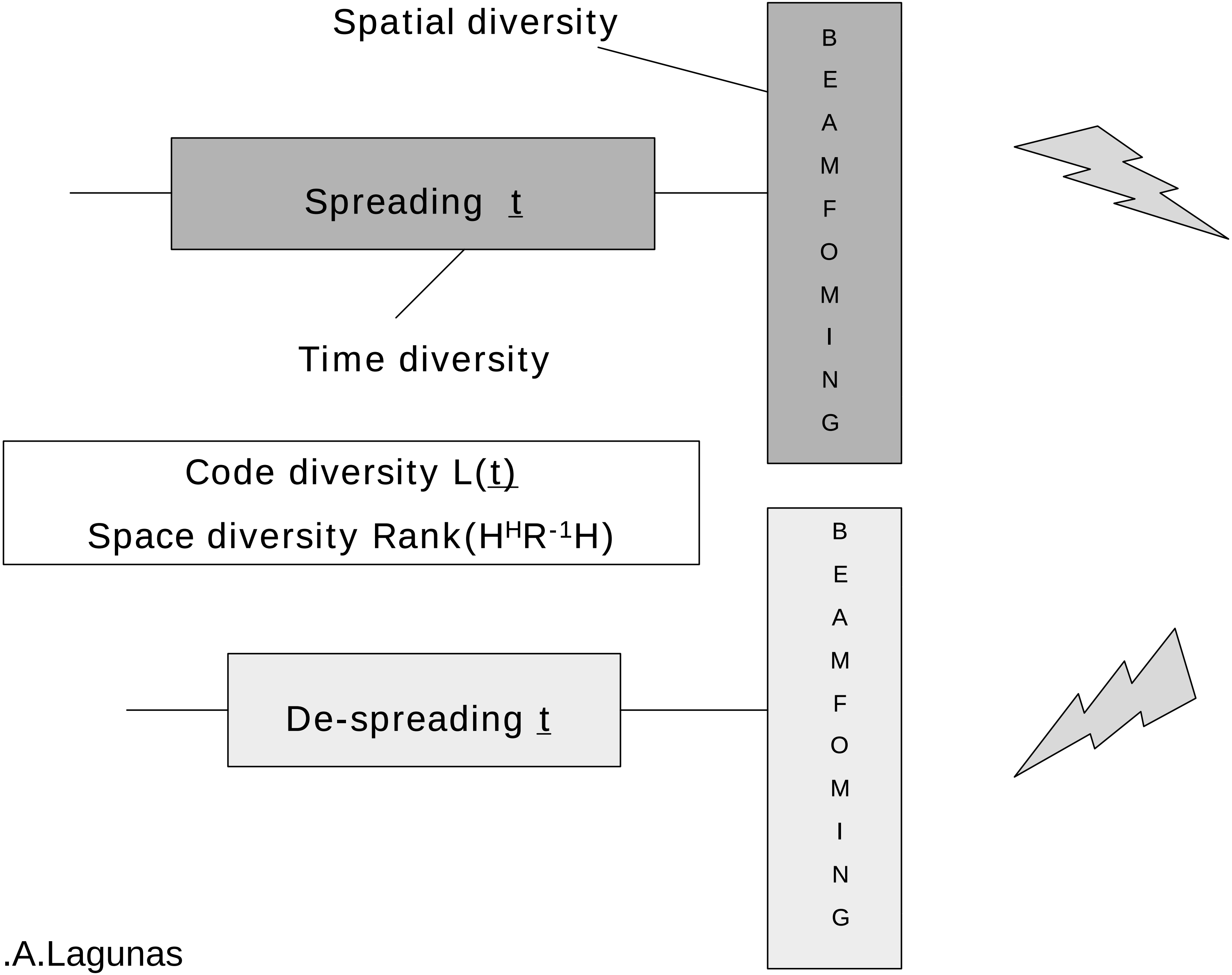
One symbol per channel use

$$SNR=\frac{\left|\underline{a}^{H}.\underline{\underline{H}}.\underline{b}\right|^{2}}{\underline{a}^{H}.\underline{\underline{R}}.\underline{a}}\leq \underline{b}^{H}.\underline{\underline{H}}^{H}.\underline{\underline{R}}^{-1}.\underline{\underline{H}}.\underline{b}\leq l_{\max}\left(\underline{\underline{R}}_{H}\right).\left|\underline{b}\right|^{2}$$

$$\underline{a}=\underline{\underline{R}}^{-1}.\underline{\underline{H}}.\underline{b}\qquad\qquad \underline{b}=\underline{e}_{\max}\left(\underline{\underline{R}}_{H}\right)$$

$$SNR=\frac{\left(Tr\left(\underline{\underline{A}}^{H}.\underline{\underline{H}}.\underline{\underline{B}}\right)\right)^{2}}{Tr\left(\underline{\underline{A}}^{H}.\underline{\underline{R}}.\underline{\underline{A}}\right)}\leq Tr\left[\underline{\underline{B}}^{H}.\underline{\underline{H}}^{H}.\underline{\underline{R}}^{-1}.\underline{\underline{H}}.\underline{\underline{B}}\right]\leq l_{\max}\left(\underline{\underline{R}}_{H}\right).Tr\left(\underline{\underline{B}}.\underline{\underline{B}}^{H}\right)$$

$$\underline{\underline{A}}=\underline{\underline{R}}^{-1}.\underline{\underline{H}}.\underline{\underline{B}}\qquad\qquad \underline{\underline{B}}=\underline{e}_{\max}\left(\underline{\underline{R}}_{H}\right).\underline{t}^{H}$$


Spatial diversity
BEAMFOMING
Spreading t
Time diversity
Code diversity L(t)
Space diversity Rank($H^H R^{-1} H$)
De-spreading t
BEAMFOMING

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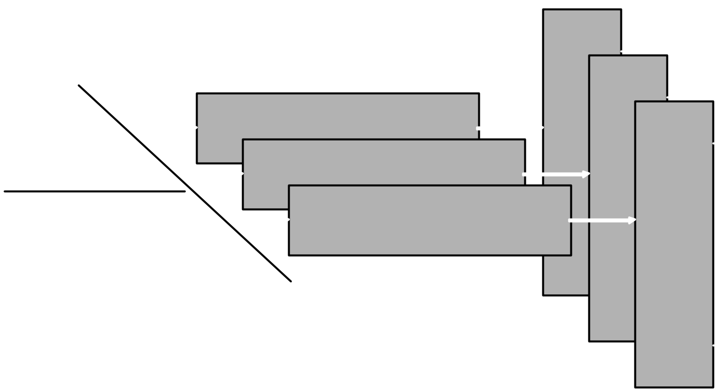
No CSIT

$$\max_{\underline{\underline{B}}}\left(\min_{\underline{\underline{R}}_H}\left(SNR\right)\right)$$

$$SNR \geq Tr\left(\underline{\underline{R}}_H\right).l_{\min}(\underline{\underline{B}}.\underline{\underline{B}}^H) \leq \frac{1}{nt}.Tr\left(\underline{\underline{R}}_H\right)$$

$$Tr\left(\underline{\underline{R}}_H\right) \geq r \qquad \underline{\underline{B}}.\underline{\underline{B}}^H = \frac{1}{nt}.\underline{\underline{I}}$$



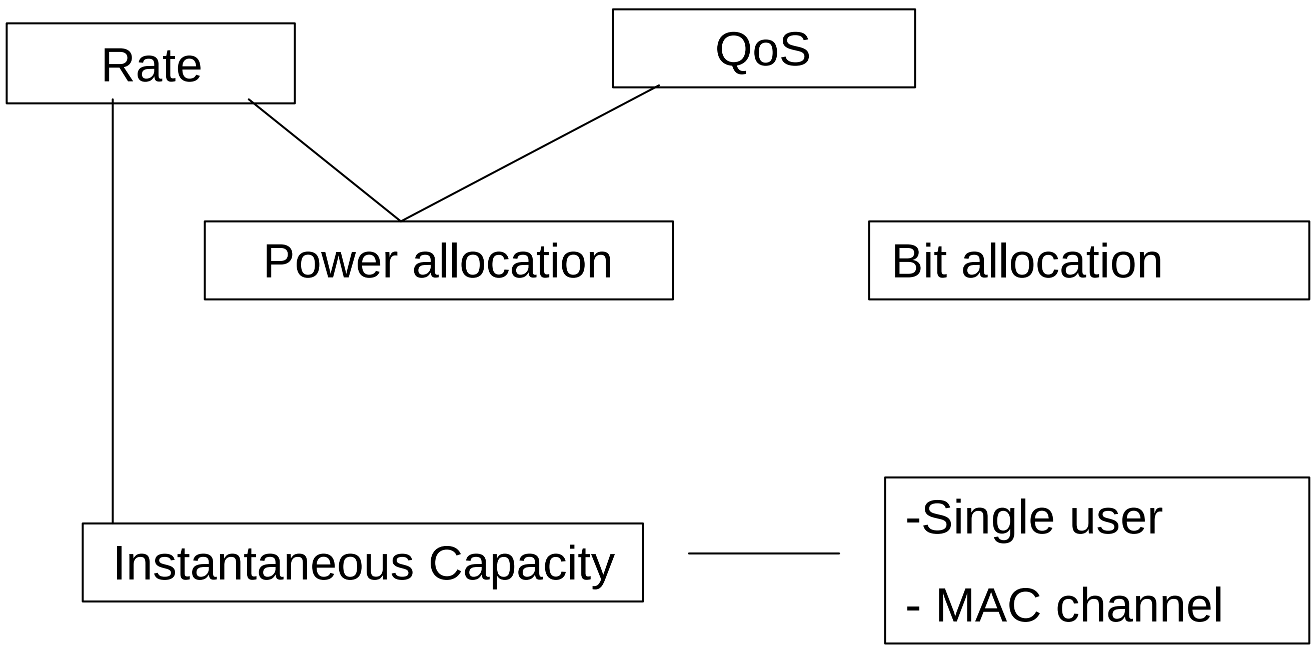
- Use of the channel eigenmodes.
- No CSI-> minmax
- Access/ OSTBC
- Full-rate and R-H family
- The goodness of uniform power allocation (UPA)

Single versus multiple beamforming

$$\underline{x}_T = \underline{\underline{B}}.\underline{x}$$

$$\underline{x}_T = \underline{b}.x$$

$$\underline{\underline{Q}} = E\left(\underline{x}_T.\underline{x}_T^H\right) = \begin{cases} \underline{\underline{B}}.\underline{\underline{B}}^H & full\ rank \\ \underline{b}.\underline{b}^H & rank\ one \end{cases}$$



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Single user instantaneous capacity

$$C = \log\det\left\lfloor \underline{\underline{I}}_{nt} + \underline{\underline{R}}_H . \underline{\underline{Q}} \right\rfloor$$

$$\max C$$
$$s.t.\ Tr\left(\underline{\underline{Q}}\right) \le P_T$$

if

$$\underline{\underline{R}}_H = \underline{\underline{U}}.\underline{\underline{Z}}.\underline{\underline{U}}^H$$

then

$$\underline{\underline{Q}} = \underline{\underline{U}}.\underline{\underline{q}}.\underline{\underline{U}}^H$$

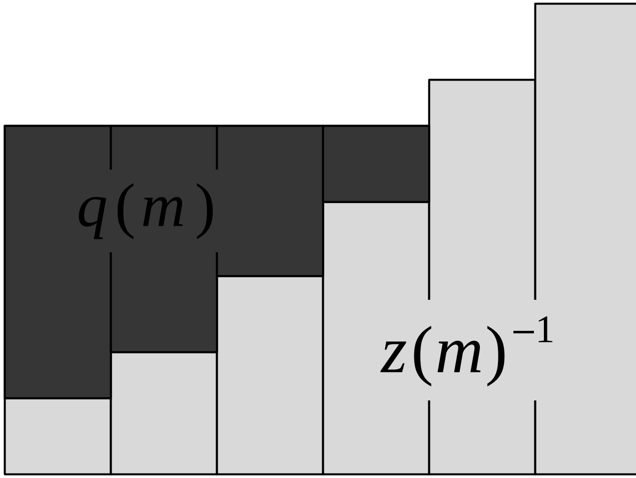
and

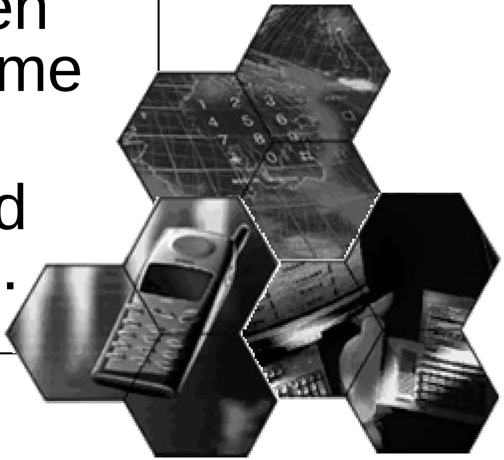
$$C = \sum_{1}^{nt} \log\left(1 + q(m).z(m)\right)$$
$$s.t.\ \sum_{1}^{nt} q(m) \le P_T$$

Optimum PA is the W-F algorithm

KKT conditions $(.)^{+}$ ———— $$q(m) + \frac{1}{z(m)} = k$$

- Single beamforming uses only the maximum (best) eigenmode.
- Under no CSIT, Uniform Power Allocation is a good (the best) choice.
- Beamforming is close to multiple beamforming (angle spread < 8º)
- UPA close for uncorrelated and even moderated correlated MIMO. Reverse W-F between channel and transmitter (Game Theory).
- Double W-F when space and frequency diversity are used.





Capacity region for the MAC channel

$$R\left(\{R_k\},\{\underline{\underline{H}}_k\}\right)=\left\{(R_1,..,R_K):0\le\sum R_k\le\log\det\left(\underline{\underline{I}}_{nR}+\underline{\underline{R}}_n^{-1}.\underline{\underline{H}}_k.\underline{\underline{Q}}_k.\underline{\underline{H}}_k^H\right)\right.$$

The capacity region is closed and convex (Verdu, Yu) and the boundary points can be found by maximizing a weighted sum of data rates

Sum-Capacity

-Solved iteratively where every user W-F considering the rest of users as interference.

- No valid for the case of beamforming, i.e. rank-one Tx.

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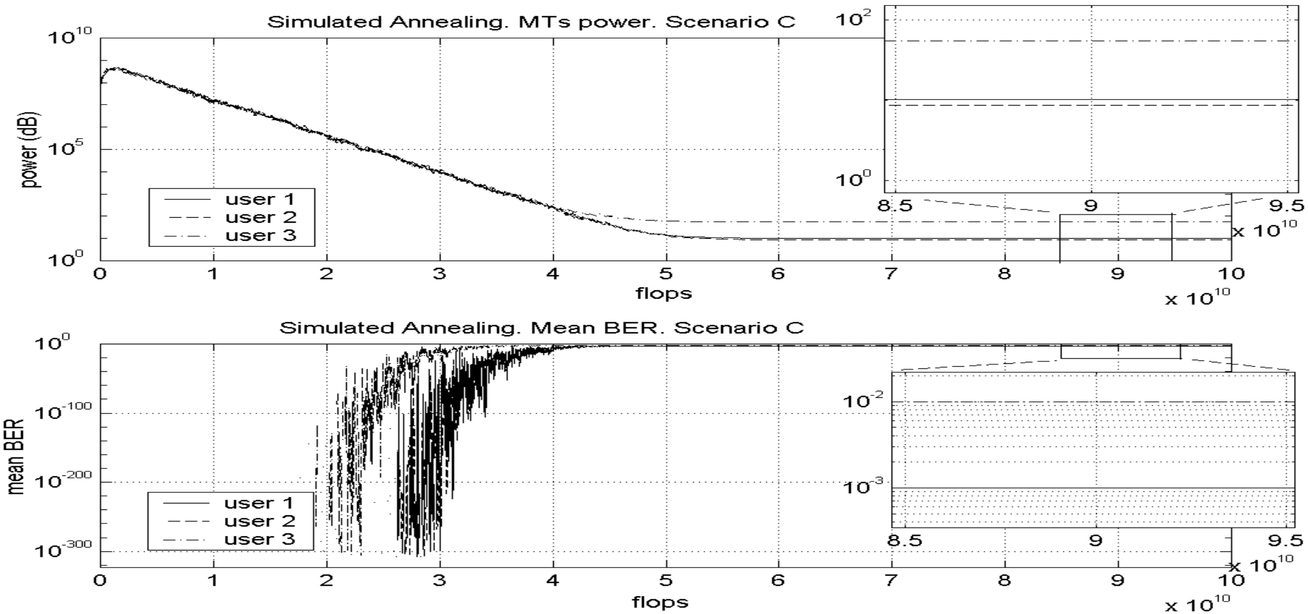
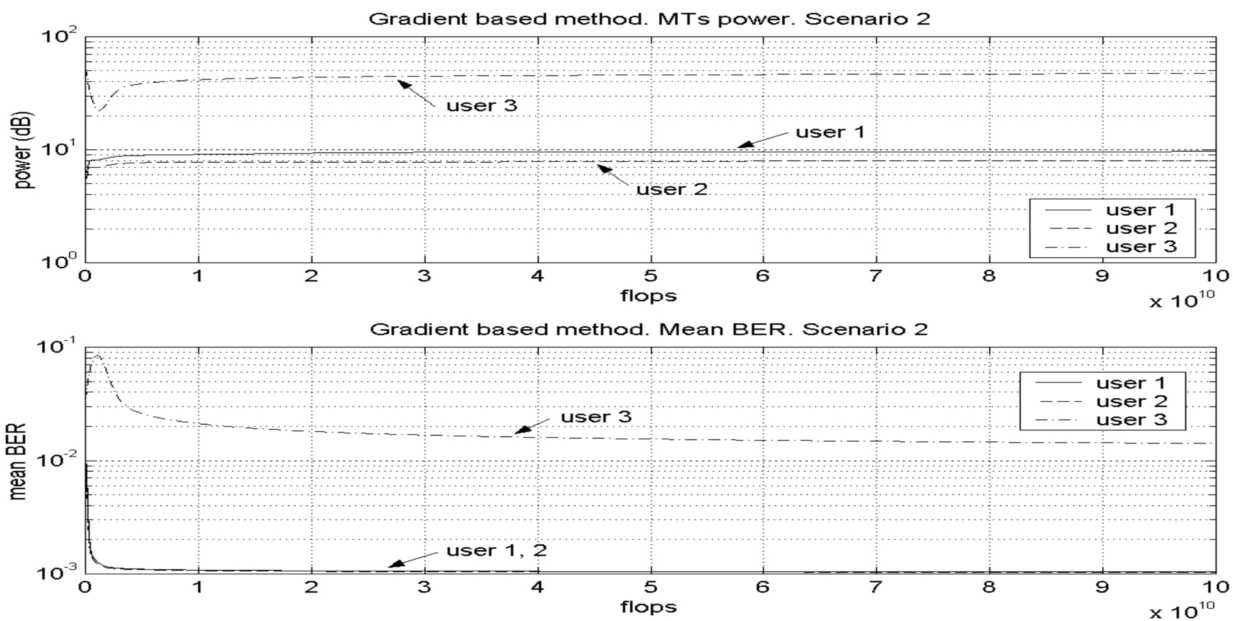


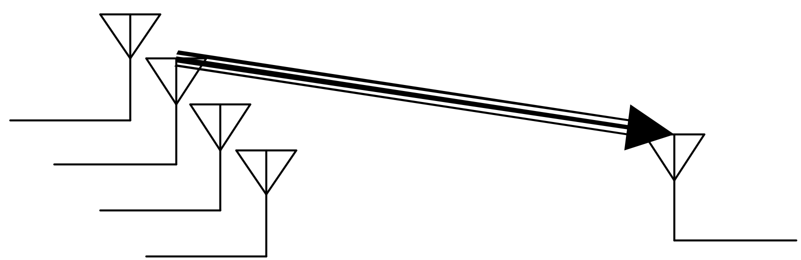
Simulated Annealing. MTs power. Scenario C
power (dB)
user 1
user 2
user 3
flops
x 10^10
Simulated Annealing. Mean BER. Scenario C
mean BER
user 1
user 2
user 3
flops
x 10^10



Gradient based method. MTs power. Scenario 2
power (dB)
user 3
user 1
user 2
user 1
user 2
user 3
flops
x 10^10
Gradient based method. Mean BER. Scenario 2
mean BER
user 1
user 2
user 3
user 3
user 1, 2
flops
x 10^10

Partial CSIT

SNR CSIT only (Adaptive modulation)→ MISO Best antenna selection



$$E_q\left[C\right]=\sum_{q=1}^{nT} Ln\left(1+\left|\underline{h}_q\right|^2 P(q)\right)$$

MIMO W-F for the nR best antennas out of nT ?

MSE Design

$$\underline{\underline{E}} = E\left[(\hat{\underline{x}} - \underline{x}).(\hat{\underline{x}} - \underline{x})^H\right] = \left(\underline{\underline{A}}^H.\underline{\underline{H}}.\underline{\underline{B}} - \underline{\underline{I}}\right)\left(\underline{\underline{B}}^H.\underline{\underline{H}}^H.\underline{\underline{A}} - \underline{\underline{I}}\right) + \underline{\underline{A}}^H.\underline{\underline{R}}.\underline{\underline{A}}$$

$$\underline{\underline{E}}(\underline{\underline{B}}) = \left[\underline{\underline{I}} + \underline{\underline{B}}^H.\underline{\underline{R}}_H.\underline{\underline{B}}\right]^{-1}$$

Possible choices

Min $f_o(MSE_{ii})$	Max $f_o(SNR_i)$	Min $f_o(BER_i)$

SOLUTION

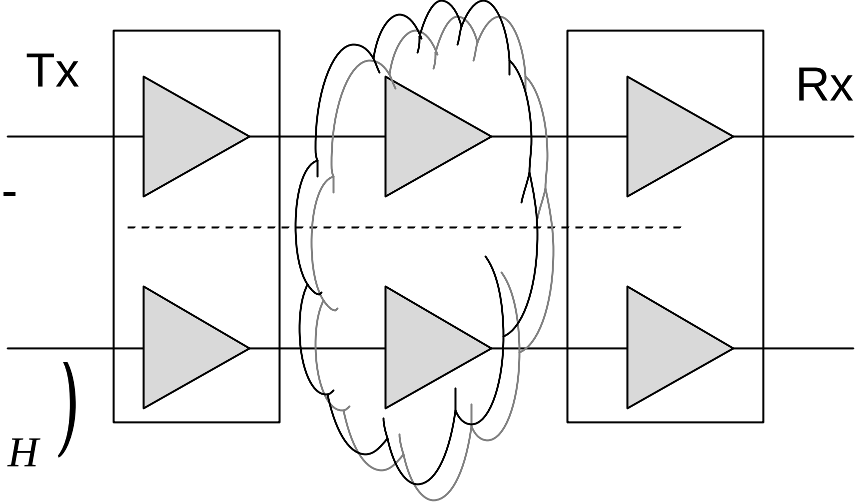
$f_o(.)$ *Schur concave* $\underline{\underline{B}} = \underline{\underline{U}}.\underline{\underline{\Sigma}}_B$

$f_o(.)$ *Schur convex* $\underline{\underline{B}} = \underline{\underline{U}}.\underline{\underline{\Sigma}}_B.\underline{\underline{V}}_B$

$\underline{\underline{V}}_B$ Unitary matrix (rotation) which transforms original MSE matrix in a new one with all the diagonal entries equal

Layered architectures are No-CSIT cases where $\underline{\underline{B}} = \underline{\underline{I}}$

Rank(R_H) Eigen-channels

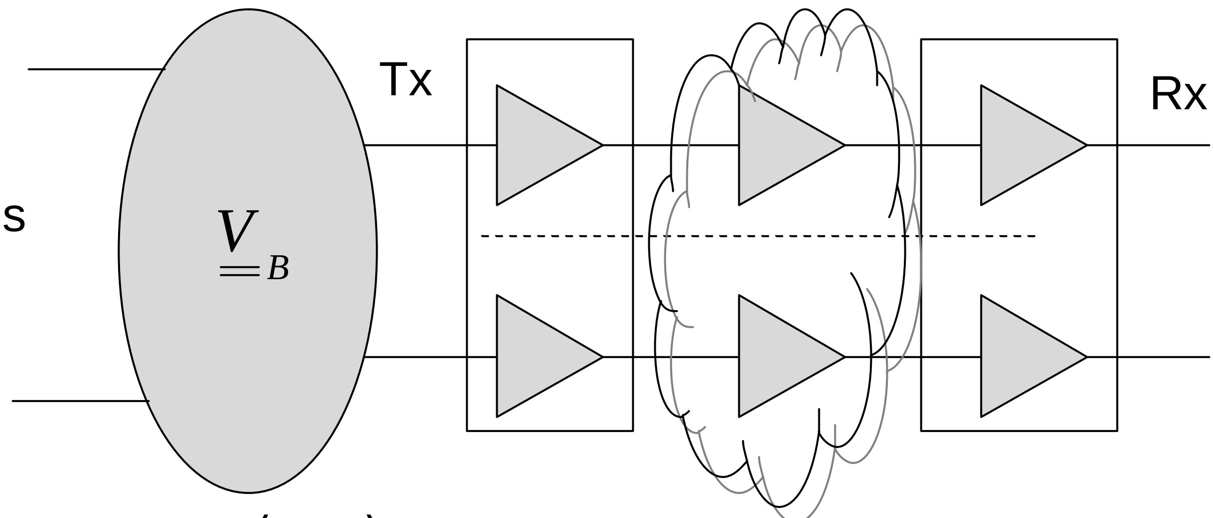


$$L \leq rank\left(\underline{\underline{R}}_H\right)$$

Concave

COOPERATIVE VERSUS NON COOPERATIVE APPROACH

symbols



Convex

$$L_o = L - Rank\left(\underline{\underline{R}}_H\right)$$

Exceeding symbols per independent channels

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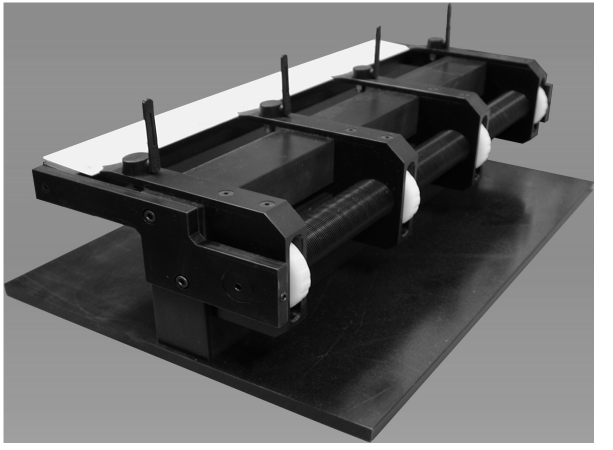
Majorization theory and Convex Optimization



$$\text{if} \quad \sum_{i=1}^{k} x(i) \le \sum_{i=1}^{k} y(i) \quad 1 \le k \le n$$

$$x(i) \ge x(i+1) \text{ and } y(i) \ge y(i+1)$$

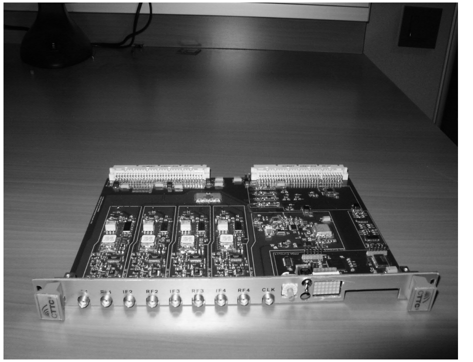
$$\text{then} \quad \underline{x} \prec \underline{y}$$

$$\text{Schur concave} \quad \text{if } \underline{x} \prec \underline{y} \quad \text{then} \quad f(\underline{x}) \ge f(\underline{y})$$

$$\text{Schur convex} \quad \text{if } \underline{x} \prec \underline{y} \quad \text{then} \quad f(\underline{x}) \le f(\underline{y})$$

$\text{Since} \quad \underline{1} \prec \underline{x}$

this reveals the "magic" of UPA in communications systems with diversity



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Arithmetic Mean MSE

$$f(MSE_{ii}) = \sum_{i=1}^{\tilde{L}} w(i).MSE_{ii}$$

$$w(i) \geq w(i+1)$$

Since the function is Concave the solution for Tx shares the eigenvectors of the channel.

$$\underline{d}(diagonal\ entries) \prec \underline{l}(eigenvalues)$$

$$f(\underline{d}) \geq f(\underline{l})$$

$$\min(f = \sum_{i=1}^{\tilde{L}} \frac{w_i}{1 + z(i).q(i)})$$

$$s.t. \quad \sum q(i) \leq P_T$$

$$\rightarrow \quad q(i) = \left[m^{-1/2}.w_i^{1/2}.z(i)^{-1/2} - z(i)^{-1} \right]^+$$

Geometric Mean MSE

$$f(MSE_{ii}) = \prod_{i=1}^{\tilde{L}} (MSE_{ii})^{w(i)}$$

$$w(i) \geq w(i+1)$$

Since the function is Concave the solution for Tx shares the eigenvectors of the channel.

$$\underline{d}(diagonal\ entries) \prec \underline{l}(eigenvalues)$$

$$f(\underline{d}) \geq f(\underline{l})$$

$$\min\left(f = \prod_{i=1}^{\tilde{L}} \left(\frac{1}{1+q(i).z(i)} \right)^{w(i)} \right)$$

$$s.t. \quad \sum q(i) \leq P_T$$

$$q(i) = \left[m^{-1}.w_i - z(i)^{-1} \right]^{+}$$

Coincides, for w(i)=1, with the capacity solution or classical W-F

Min-Max MSE

$$f\,(MSE_{ii}) = \max(MSE_{ii})$$
$$i = 1 : \tilde{L}$$

Since the function is Convex the solution for Tx shares the eigenvectors of the channel up to a rotation to obtain equal entries on the MSE matrix.

$$\min\ t$$
$$s.t.\quad t \geq \frac{1}{L}.\left[L_o + \sum_{i=1}^{\tilde{L}} \frac{1}{1+q(i).z(i)}\right] = \frac{1}{L} Tr\left(\underline{\underline{E}}\right) = MSE_{ii}$$
$$\sum q(i) \leq P_T$$

μ Selected to hold with equal sign the constrains

$$q(i) = \left[m^{-1/2}.z(i)^{-1/2} - z(i)^{-1}\right]^{+}$$

Suboptimum solution. To impose diagonality (non-cooperative transmission)

$$q(i) = z(i)^{-1}.m$$

Direct logarithmic W-F (opposed to the max capacity solution

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Geometric Mean SNR

$$f(SNR_i) = \prod_{i=1}^{\tilde{L}} (SNR_i)^{w(i)}$$

$$w(i) \leq w(i+1)$$

Since the function is S-Concave the solution for Tx shares the eigenvectors of the channel.

$$\underline{d}(diagonal\ entries) \prec \underline{l}(eigenvalues)$$

$$f(\underline{d}) \geq f(\underline{l})$$

$$\min\left[f = \sum_{i=1}^{\tilde{L}} \left(q(i).z(i)\right)^{w(i)} \right]$$

$$s.t. \quad \sum q(i) \leq P_T$$

$$q(i) = w(i).P_T$$

Uniform Power Allocation

Equal to max capacity for no CSIT

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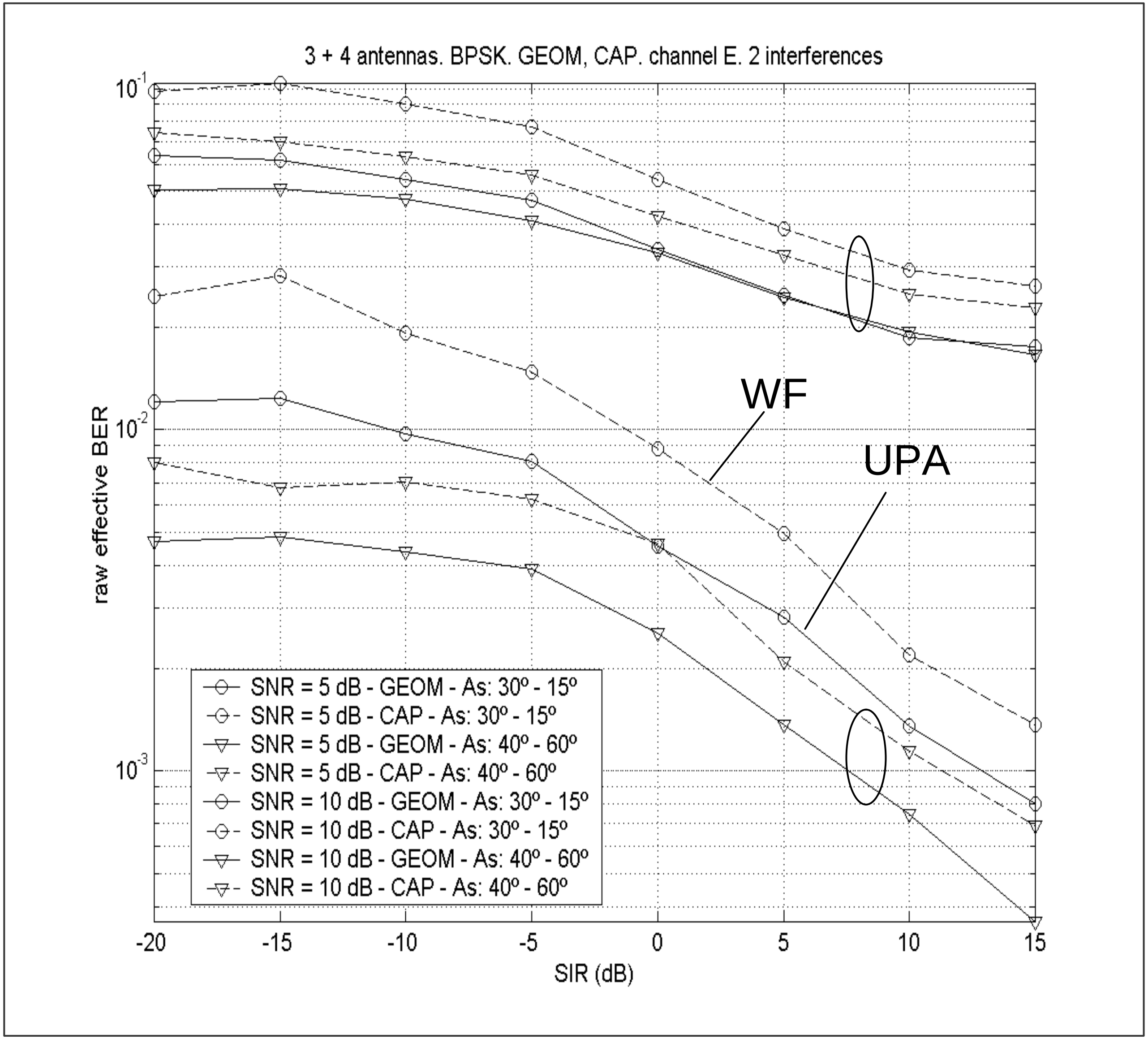
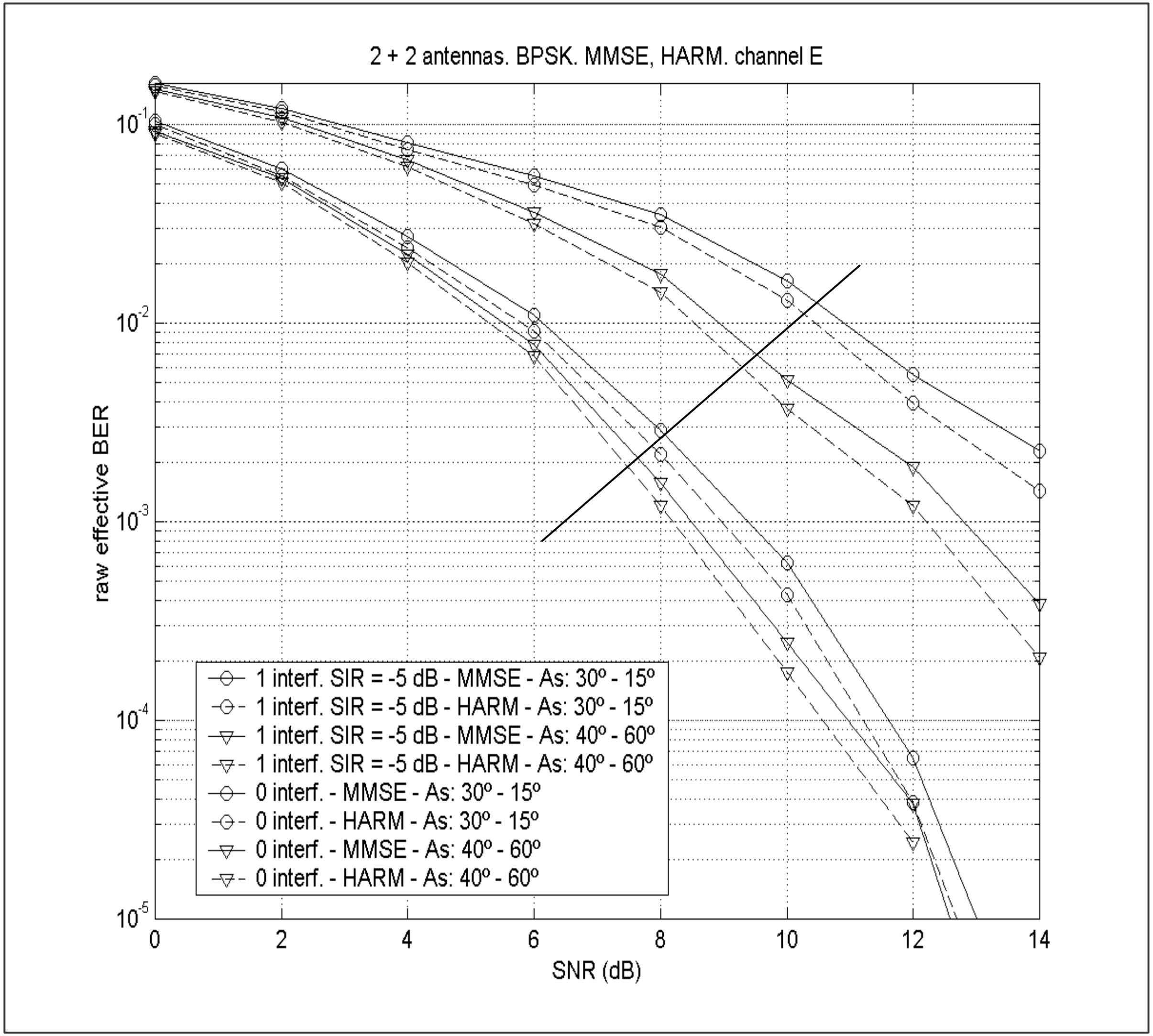


- ***GEOM vs CAP***
- ***3+4 antennas***
- ***Channel E (250 ns)***
- ***2 interferences***
- ***SNR (5, 10 dB)***
- ***Angular***
- ***spread (TX-RX):***

  30º - 15º

  40º - 60º
- ***BER vs SIR***

UPA better than the W-F. The same for high SNR and uncorrelated MIMO.



- ***MMSE vs HARM***
- ***2+2 antennas***
- ***Channel E (250 ns)***
- ***0 interferences***
- ***1 interference***

  (SIR = -5 dB)
- ***Angular spread (TX-RX):***

  30º - 15º

  40º - 60º
- ***BER vs SNR***

Closeness of arithmetic MSE (Complex WF) and Harmonic mean of the SNR (Logarithmic WF)

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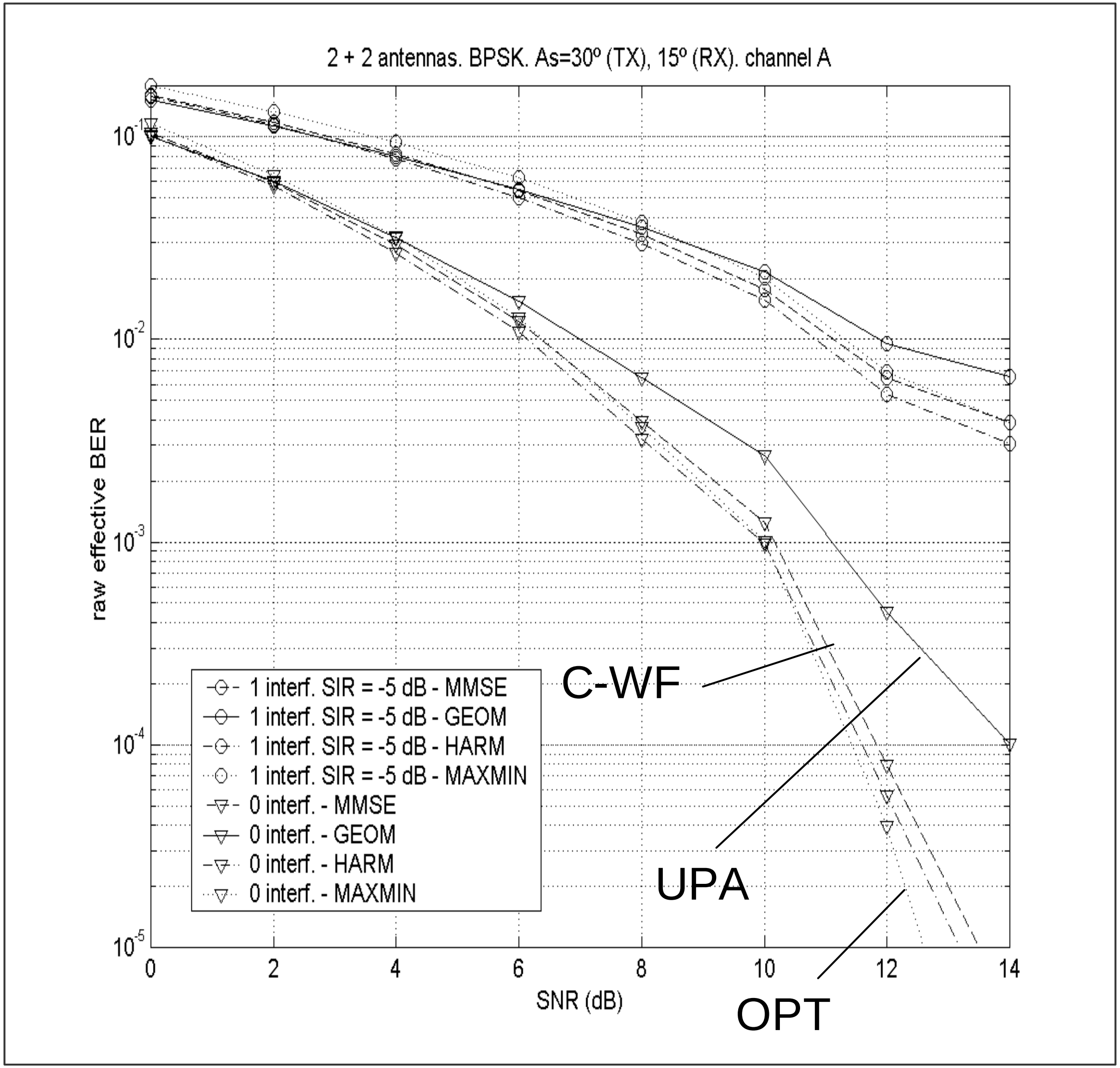

- ***2+2 antennas***
- ***Channel A (50 ns)***
- ***0 interferences***
- ***1 interference (SIR = -5 dB)***
- ***Angular spread (TX-RX): 30º - 15º***
- ***BER vs SNR***

Closeness of Arithmetic MSE, Harmonic Mean SNR and Minimax

- Aritm MSE close to MinMax MSE
- Aritm SNR. SConcave. All to best mode→ very low spectral efficiency.
- Harmonic mean SNR. Sconvex. Optimal solution equal to MinMax MSE. Suboptimal produces inverse log WF.
- Geometric mean SNR reduces to Uniform Power Allocation.
- Geometric mean BER. All to best mode.
- Arithmetic BER. Optimum performance with not closed form.
- Closeness of Arit MSE, Harm SNR and MaxMin
- MinMax of MSE, MaxMin SNR and MinMax BER are equal.